Axiomatic Truth

Attempts to define truth (as correspondence, or coherence, or in some pragmatic way) have not produced anything very precise, and many philosophers were dissatisfied. Logicians and mathematicians needed a precise account of truth, in order to characterise reasoning in terms of the way in which truth is transmitted. From this point of view, it appeared that it was impossible for a language to contain its own well-defined truth predicate, and so the strategy was to step outside the language, and say that a sentence is true in some language if it is endorsed in a 'meta-language', which describes what the 'object language' can do. This semantic or disquotational account of truth met the needs of formal discussion, but just seemed to offer a vast list of sentences which could be 'endorsed' in language, without saying what 'true' meant. 'True' is not even part of the bottom level object language. Then truths in the meta-language had to be endorsed in a 'meta-meta-language', leading to an endless hierarchy of nested languages (a 'typed' theory), each specifying the truths in the language below it. The new formal theory also relied on the idea of 'satisfaction', which is a two-place predicate ('A satisfies B'), whereas truth is a one-place predicate ('A is true'). Hence a working system was now available, but we were little wiser about what 'true' really amounted to.

Fans of this disquotational approach are inclined to say that truth in everyday language is either redundant, or has a minimal role. If 'is true' adds nothing to a sentence then it is redundant, and if it just facilitates reference to other sentences, or generalisations about them, then its role is minimal. If these views also seem inadequate to capture 'truth', then we seem to need a theory which includes the truth predicate in our basic object language, but then give it a stronger role, preferably expressed in a fairly precise way. Enter the axiomatic theory. The first step is to avoid the difficulty of defining 'true' in our bottom level language, by treating it (initially, at least) as a primitive, that is, an acceptable concept which starts off with no content. The content is then given by the role of truth in language, rather than its intrinsic nature, and this role is expressed by a set of axioms. We usually approach ethics by trying to understand the essential nature of 'good' or 'right'. Set theory, on the other hand, just introduces the relation 'is a member of', but says nothing about what this means, and gives its entire behaviour in the famous set theory axioms. The latter approach offers to reveal very precise details about how we understand truth, and to offer us different axiom systems to capture different understandings.

It would be nice if there was a quick consensus about the axioms which generate just the normal behaviour of 'true' that we all agree on, but this does not happen. Because it is possible, using a numbering system, to express linguistic reasoning in arithmetical terms, and because arithmetic is precise and well-defined, standard arithmetic is used as the test for the axiomatic systems. The question for each proposed axiom system is how much can be thereby proved in arithmetic. Axioms are described as 'weak' or 'strong', depending on how much they can prove. Any theory should probably be 'conservative', meaning that the addition of 'true' to the language of arithmetic does not facilitate any extra proofs. Standard arithmetic is described as 'categorical', meaning that all versions derived from its axioms will map onto one another, and so it is important to see whether adding 'true' to arithmetic will undermine this attractive feature. Do we also want to say that truths are true (and so on), and tweak the axioms to make this possible?

The original assumption of axiomatic theories was to rely on classical logic, which assumes that every proposition is either T or F. We may, however, want a view of truth which allows 'gaps' (e.g. T, F and U, for 'unknown'), or even 'gluts' (treating U as a sort of truth, so that T and F overlap), and the axioms will need adjustment to express these ideas. An aspiration of some axiomatic theories is to allow the object-language to once again contain its own truth predicate (which had been ruled out in the hierarchies of the original typed theory). 'True' had been banned because of the Liar Paradox, but it is now thought that a 'Strengthened Liar' paradox makes inclusion of 'true' within your language very difficult. If we say 'this sentence is neither T nor F, or it is not T', this either reduces to the ordinary Liar, or it generates the paradox in the case of a 'gap'. It may be that only hierarchical axiomatic theories will do the job we want. This is an area of research among current logicians, and there is no consensus on the best system of axioms.

The earliest axiomatic theory took the huge list that made up the definition of the truth, and treated each item as an axiom. The resulting system (NT) was weak, but reached basic standards of adequacy. However, it is not fully consistent, and even permits the Liar paradox, so it won't do.

An intermediate theory (CT) treats truth as 'compositional', with desirable rules for how truth for sentences is built up from the truth of its ingredients. A much better theory (FS) was formulated using this basis, but without the strict rules for a typed hierarchy. The new theory was conservative (unlike CT), and key axioms were added, saying that if a sentence is assigned 'T', you can then infer that sentence, and also that if you assert a sentence, an axiom allows you to attach 'T' to it. This connects the concept of 'true' to our ordinary notions of being allowed (or forbidden) to assert some sentence.

Another major theory (KF) explored the possibility of using three-valued Strong Kleene logic, in order to deal with potential gaps or gluts in truth values. The result is a much stronger system (able to prove much more), but an aim of the theory was a language which contained its own truth predicate (which is forbidden in earlier systems, but allowed in most natural languages such as English), and it doesn't quite achieve this. There are many other axioms systems, each with its variants, and there is at least some convergence on what a good theory should look like. The theories will never solve all of the problems, because it has been proved that they can never be formally 'complete'.

Clearly these issues must be left to experts, but their investigations are important for philosophy. Before formal systems of truth were investigated, the whole concept of truth was falling into disrepute, and so one of the pillars of philosophy appeared to be crumbling, with major consequences for the subject. No one thinks 'true' is a forbidden word, only usable if it has secure technical foundations, but the modern dream of making philosophy a rigorous subject seems to depend on whether its technicians can produce a satisfactory theory of how 'true' works. There might even eventually emerge a simple account of the best theory, accessible for ordinary speakers of languages.